Dividing Radical Expressions

QUOTIENT PROPERTY OF RADICAL EXPRESSIONS

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, $b \neq 0$, and for any integer $n \geq 2$ then,

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad \text{and} \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Simplify:

ⓐ $\dfrac{\sqrt{72x^3}}{\sqrt{162x}} = \sqrt{\dfrac{72x^3}{162x}}$

$= \sqrt{\dfrac{36x^3}{81x}}$

$= \sqrt{\dfrac{36x^2}{81}} = \sqrt{\dfrac{4x^2}{9}}$

$= \dfrac{6x}{9} = \boxed{\dfrac{2x}{3}}$

ⓑ $\dfrac{\sqrt[3]{32x^2}}{\sqrt[3]{4x^5}} = \sqrt[3]{\dfrac{32x^2}{4x^5}}$

$= \sqrt[3]{\dfrac{8}{x^3}}$

$= \dfrac{2}{x}$

Simplify:

ⓐ $\dfrac{\sqrt{50s^3}}{\sqrt{128s}} = \sqrt{\dfrac{50s^3}{128s}}$

$= \sqrt{\dfrac{25s^2}{64}}$

$= \dfrac{5s}{8}$

ⓑ $\dfrac{\sqrt[3]{56a}}{\sqrt[3]{7a^4}}. = \dfrac{2}{a}$

$= \sqrt{\dfrac{8}{a^3}}$

$= \dfrac{2}{a}$

Simplify:

$$2-(-1)=3$$
$$-1-2=-3$$

$$\frac{\sqrt{162x^{10}y^2}}{\sqrt{2x^6y^6}} = \sqrt{\frac{162x^{10}y^2}{2x^6y^6}}$$
$$= \sqrt{\frac{81x^4}{y^4}}$$
$$= \frac{9x^2}{y^2}$$

$$\frac{\sqrt[3]{-128x^2y^{-1}}}{\sqrt[3]{2x^{-1}y^2}} = \sqrt[3]{\frac{-128x^2y^{-1}}{2x^{-1}y^2}}$$
$$= \sqrt[3]{\frac{-64x^3}{y^3}}$$
$$= \frac{-4x}{y}$$

Simplify:

$$\frac{\sqrt{300m^3n^7}}{\sqrt{3m^5n}}$$

$1-(-2)=3$

$-1-5=-6$

$$\frac{\sqrt[3]{-81pq^{-1}}}{\sqrt[3]{3p^{-2}q^5}} = \sqrt[3]{\frac{-81pq^{-1}}{3p^{-2}q^5}}$$

$$= \sqrt[3]{\frac{-27p^3}{q^6}}$$

$$\frac{-3p}{q^2}$$

Simplify:

$$\frac{\sqrt{54x^5y^3}}{\sqrt{3x^2y}}.$$

$$\frac{\sqrt{64x^4y^5}}{\sqrt{2xy^3}}.$$

RATIONALIZING THE DENOMINATOR

Rationalizing the denominator is the process of converting a fraction with a radical in the denominator to an equivalent fraction whose denominator is an integer.

SIMPLIFIED RADICAL EXPRESSIONS

A radical expression is considered simplified if there are

- no factors in the radicand have perfect powers of the index
- no fractions in the radicand
- no radicals in the denominator of a fraction

Simplify:

$\sqrt{20} = \sqrt{4} \cdot \sqrt{5}$

$2\sqrt{5}$

$$\frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{4\sqrt{3}}{\sqrt{9}}$$

$$\frac{4\sqrt{3}}{3}$$

$$\sqrt{\frac{3}{20}} = \frac{\sqrt{3}}{\sqrt{20}}$$

$$\frac{\sqrt{3}}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$\frac{\sqrt{15}}{2 \cdot 5}$$

$$\frac{\sqrt{15}}{10}$$

$$\frac{\sqrt{3}}{\sqrt{20}} \cdot \frac{\sqrt{20}}{\sqrt{20}}$$

$$\frac{\sqrt{60}}{20}$$

$$\frac{\sqrt{4} \cdot \sqrt{15}}{20}$$

$$\frac{2\sqrt{15}}{20}$$

$$\frac{3}{\sqrt{6x}} \cdot \frac{\sqrt{6x}}{\sqrt{6x}}$$

$$\frac{\overset{1}{\cancel{3}}\sqrt{6x}}{2\cancel{6}x}$$

$$\frac{\sqrt{6x}}{2x}$$

Simplify:

$$\frac{1}{\sqrt[3]{6}} \cdot \frac{\sqrt[3]{36}}{\sqrt[3]{36}}$$

$$\frac{\sqrt[3]{36}}{\sqrt[3]{216}}$$

$$\frac{\sqrt[3]{36}}{6}$$

$$\sqrt[3]{\frac{7}{24}}$$

$$\frac{\sqrt[3]{7}}{\sqrt[3]{24}} \quad \frac{\sqrt[3]{24^2}}{\sqrt[3]{24^2}}$$

$$\frac{\sqrt[3]{4032}}{24}$$

$$\sqrt[3]{8} \cdot \sqrt[3]{504}$$

$$\frac{2\sqrt[3]{504}}{24}$$

$$\frac{\sqrt[3]{504}}{12}$$

$$\frac{2\sqrt[3]{63}}{12}$$

$$\frac{\sqrt[3]{63}}{6}$$

$$\frac{\sqrt[3]{7}}{\sqrt[3]{24}}$$

$$\frac{\sqrt[3]{7}}{2\sqrt[3]{3}} \cdot \frac{\sqrt[3]{9}}{\sqrt[3]{9}}$$

$$\frac{\sqrt[3]{63}}{2 \cdot \sqrt[3]{27}}$$

$$\frac{\sqrt[3]{63}}{6}$$

$(4x)(4x) = 16x^2$

$$\frac{3}{\sqrt[3]{4x}} \cdot \frac{\sqrt[3]{16x^2}}{\sqrt[3]{16x^2}}$$

$$\frac{3\sqrt[3]{16x^2}}{\sqrt[3]{64x^3}}$$

$$\frac{3\sqrt[3]{16x^2}}{4x}$$

Simplify:

$$\frac{1}{\sqrt[3]{7}} \cdot \frac{\sqrt[3]{49}}{\sqrt[3]{49}}$$

$$\frac{\sqrt[3]{49}}{7}$$

$$\sqrt[3]{\frac{5}{12}} = \frac{\sqrt[3]{5}}{\sqrt[3]{12}} \cdot \frac{\sqrt[3]{144}}{\sqrt[3]{144}}$$

$$= \frac{\sqrt[3]{720}}{12}$$

$$= \frac{\sqrt[3]{8} \cdot \sqrt[3]{90}}{12}$$

$$\frac{2\sqrt[3]{90}}{12}$$

$$\frac{\sqrt[3]{90}}{6}$$

$$\frac{5}{\sqrt[3]{9y}} \cdot \frac{\sqrt[3]{81y^2}}{\sqrt[3]{81y^2}}$$

$$\frac{5\sqrt[3]{81y^2}}{9y}$$

Simplify:

$$\frac{1}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{8}}{\sqrt[4]{8}}$$

$$\frac{\sqrt[4]{8}}{2}$$

247, 251

255, 257

261, 265, 266

$$\sqrt[4]{64} = \sqrt[4]{16} \cdot \sqrt[4]{4}$$

$$2\sqrt[4]{4}$$

$$\sqrt[4]{\frac{5}{64}} = \frac{\sqrt[4]{5}}{\sqrt[4]{64}}$$

$$= \frac{\sqrt[4]{5}}{2\sqrt[4]{4}} \cdot \frac{\sqrt[4]{4^3}}{\sqrt[4]{4^3}}$$

$$\frac{\sqrt[4]{320}}{2 \cdot 4}$$

$$\frac{2\sqrt[4]{20}}{8}$$

$$\frac{\sqrt[4]{20}}{4}$$

$$\frac{2}{\sqrt[4]{8x}} \cdot \frac{\sqrt[4]{8^3x^3}}{\sqrt[4]{8^3x^3}}$$

$$\frac{2\sqrt[4]{512x^3}}{8x}$$

$$\frac{2 \cdot 4\sqrt[4]{2x^3}}{8x}$$

$$\frac{\sqrt[4]{2x^3}}{x}$$